

Maneuver Prediction and Planning for Automated and Connected Vehicles based on Interaction and Gaming Awareness under Uncertainty

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Abstract: The complex and mixed traffic environment makes it a challenge for the widespread use of automated and connected vehicles (ACVs). It is necessary for these systems to have a better understanding of the traffic environment including interaction and gaming between multiple vehicles. In this study, a maneuver prediction and planning framework is proposed on the basis of game theories for complex and mixed traffic scenarios via Vehicle-to-Everything (V2X) communication. In this framework, the interaction and gaming between multiple vehicles are considered by employing the extensive form game theories. In the payoff function, the risk assessment model based on trajectory prediction under uncertainty is employed to assess collision risks. Driving efficiency and preference are also combined in the payoff function. Uncertainty elements, including estimation and prediction, are considered to predict and plan by using Nash equilibrium of the extensive form game theory in mixed and behavioral strategies. Finally, this framework is applied and proved in different lane-change scenarios. The results show that this framework could predict other vehicles' driving maneuvers and plan maneuvers for ego vehicles by considering interaction and gaming between multiple vehicles, which helps ACVs understand the environment better and make the cooperative maneuver planning in complex traffic scenarios.

1 Introduction

ACVs have received extensive research interest because they show great potential for use in more efficient, safer, and cleaner transportation systems [1]. Developments in this field will evidently increase in both quality and importance over time [2]. However, the problem is why Automated Vehicles (AVs) are not able to be widely used at present in the complex and mixed traffic, which includes other road users such as human drivers. Technically speaking, one of the crucial reasons is that the traffic environment is increasing complex as well as full of uncertainty even with V2X communication [3]. To solve this problem, as human driving skills, AVs should have the ability to predict changes of the traffic environment under uncertainty, such as the trajectory of other vehicles. Also, the strategies of surrounding vehicles are influenced with each other. Therefore, it is crucial for AVs to have a better understanding of the traffic environment including interaction and gaming under traffic estimating and predicting uncertainty via V2X communication.

Situational awareness (SA) is one of the indispensable parts for AVs to understand the environment, especially in complex traffic scenarios, as shown in Fig. 1. The work of SA is to percept the elements in the environment, comprehend their meaning and project their status in the near future, which was considered in [4]. Like human drivers, the traffic environment prediction, which helps make right decisions, plays crucial roles in SA. In addition, other vehicles in the traffic influence the prediction and decision making of the ego vehicle and decisions of the ego vehicle also have impacts on traffic environments such as other vehicles' behaviors. And they all are aware of this, which brings out the interaction and gaming between multiple vehicles. The basic difficulty of the interaction and gaming is a circular concept: a player in the traffic makes the best decision against the conjectured strategies of the other players in the traffic but, in turn, this best decision and planning should be conjectured by

the other players in the traffic and they should play the best replies as well [6]. Moreover, the elements of uncertainty are common in the traffic including observation and prediction. As a result, the uncertainty should be considered in SA, as shown in Fig. 1. With the development of communication technology [7] [8], V2X communication devices are employed to obtain much more information to predict and plan in the complex traffic environment. In this proposed framework for maneuver prediction and planning, environment prediction, multiple vehicle interaction and gaming, and uncertainties are considered to improve SA.

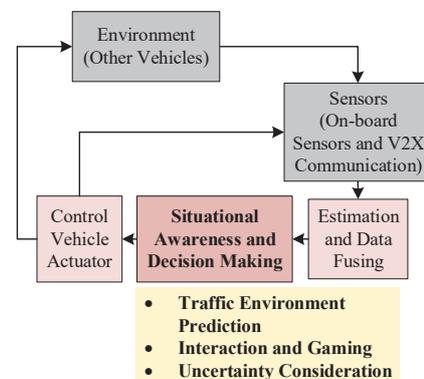


Fig. 1: The structure of an intelligent vehicle including SA and decision making and SA has three main features namely traffic environment prediction, interaction and gaming as well as uncertainty consideration

The maneuver prediction and planning is to predict and plan simultaneously at a high level. The maneuver planning in this study is about High-level controller (HLC) to design the finite state machine (FSM), which was proposed and studied in [9]. Then trajectory planning and tracking models could be used to realize the automated functions [10][11]. Since the prediction of the environment and planning of the ego vehicle influence with each other, prediction and planning should be considered at the same time. Meanwhile, surrounding vehicles also try to make the best response rationally. In other words, they interact and game with each other, which was not considered much as in [12]. As an example shown in Fig. 2, Vehicle B is approaching Vehicle D in the middle lane. As a result, Vehicle B would make a decision to change the lane for better traffic environment or less payoff cost. In addition, Vehicle B has to consider whether Vehicle A would accelerate heavily to prevent Vehicle B changing to the left lane or not. Furthermore, the maneuvers of surrounding vehicles (Vehicle A, Vehicle D and Vehicle C) will be influenced according the strategy of Vehicle B.

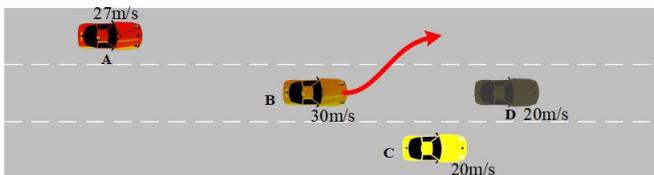


Fig. 2: One scenario of lane changing with interaction and gaming between multiple vehicles

There are various methods to deal with motion predictions for ACVs such as physics-based models [6], machine learning methods [13][19][22], and game theories [30]. Physics-based models are based on vehicle kinematic or dynamic models, which assume that some parameters stay constant. Physics-based models could only predict at a low level with a short term. Machine learning methods could learn knowledge from data by combining multiple parameters nonlinearly. However, Machine learning methods ignored the gaming of multiple vehicles in the surrounding environment. In addition, game theories have been also employed to deal with the gaming between multiple vehicles. But the uncertainty in the process of gaming and multi-act games with much longer horizon have been seldom researched. In this study, the motion prediction under uncertainty, and interaction and gaming between multiple vehicles are considered to predict and plan maneuvers at the same time based on extensive form games.

The objective of this study is to build a framework of maneuver prediction and planning, namely left lane-change, right lane-change and lane keeping, for a few steps simultaneously. In this framework, the maneuver prediction for other vehicles and maneuver planning for ego vehicle are considered on the basis of the interaction and gaming between multiple vehicles in traffic by employing extensive form game theories, which have been extensively researched for sequential gaming problems. And it is assumed that each player in traffic is rational and seeking for the minimized payoff cost. In the payoff function, the risk assessment model based on environment estimation and prediction under uncertainty, which integrates the dynamic and maneuver-based predicting models for accurate results in the long term [5], is employed to assess collision risks. Moreover, considering the uncertainty of traffic environments, the payoff function in this study combines risk assessments, driving efficiency and preference. Based on the consideration of other vehicles' strategies at the same time, the planning results for the ego vehicle could show the cooperative decision-making strategies. Also, the multi-step prediction and planning will be studied, which could be aware of the potential risks and respond earlier.

The remainder of this paper is organized as follows: Section 2 introduces the related work pertaining to maneuver and motion prediction. Section 3 presents the methodology used in this study, which includes the introduction of the extensive form game theory, risk

assessment based on vehicle trajectory prediction as well as the payoff function for the interaction and gaming. And the application of the framework in different lane-change scenarios and the results are described and analyzed in Section 4. At last, Section 5 presents some conclusive remarks.

2 Related Work

As mentioned previously, maneuver prediction and planning are crucial for the research of ACVs. Therefore, many research efforts have been expended on motion prediction and planning for ACVs using various methods such as physics-based models [6], machine learning methods [19][22], and game theories [30].

Physics-based models are the widespread techniques to predict the motions, which consider the motion prediction based on physics laws and assume that some parameters such as velocity, acceleration, and yaw rate are constant [14][15][16][17]. Huang J, et al. [14] explored vehicle future trajectory prediction and examined the possible methodologies for trajectory prediction based on differential global positioning system. In this study, different dynamic trajectory prediction models were compared; results indicated that one of the main error causes was the changes in driver's intention. Sorstedt J, et al. [15] considered the driver control input parameter to obtain better predictions. Polychronopoulos, Aris, et al. proposed a model-based description of the traffic environment for an accurate prediction of vehicle path, which was situation adaptive and calculation dynamic [16]. In the study, dynamic models such as constant acceleration (CA), constant turn rate (CTR), and constant turn rate constant tangential acceleration (CTRA) models were included for the accurate path prediction. However, physics-based models can only predict with sustainable results in the short term. They could not predict at a higher level. In addition, physics-based models are not able to take the multiple vehicles' interaction and gaming into consideration.

Machine learning methods have the ability to solve the nonlinear problems in the research of motion prediction, which have been widely used to predict vehicle paths and estimate maneuvers by learning from driving data [18][19][20]. Hou, et al. developed a model to predict driver decisions on whether to merge or not as a function of certain input variables [18], which was on the basis of Bayesian network and decision trees. In [19], a neural network-based model was developed and studied in detail to recognize the lane-change behaviors and predict the lane-change trajectory. Peng, et al. [20] mentioned a back-propagation neural network model to predict lane-changing behavior. In the study, the lane-change-intent time window was designed approximately as 5 seconds. Machine learning methods could learn the knowledge from driving data, which are able to combine several parameters nonlinearly. However, these methods did not take the traffic sequential information into account and they failed to simulate the interaction and gaming between multiple vehicles.

By considering the sequential information, some approaches including dynamic Bayesian network [21], and Markov theories were employed to estimate motions and driving maneuvers [22][23][24][26][27]. Li, et al. [22] proposed a novel algorithm combining the hidden Markov model (HMM) and Bayesian filtering (BF) techniques to estimate a lane-changing maneuver. In this study, the grammar definition was inspired by speech recognition models based on Markov theories. In [25], the sequence prediction method for driving behavior is obtained using a nested Pitman-Yor language model (NPYLM), which was originally proposed in the natural language processing field. In [26] [27], the authors built the maneuver recognition and prediction model based on data learning such as hidden Markov models (HMM). Li, et al. proposed models to infer driving intentions by considering the impact of past driving behavior on current station with add Auto-regression (AR) [23]. Gindele, et al. [24] modeled the decision-making process of drivers by building a hierarchical dynamic Bayesian model that described physical relationships as well as the driver's behaviors and plans, which aimed to estimate and predict traffic situations over time. These methods

estimated the motions in the maneuver level and considered the interaction between vehicles. However, gaming between vehicles was failed to consider.

Game theories have been extensively researched in a variety of areas for multi-agent interaction problems including traffic environment prediction for AVs [28][29][30]. Talebpour, et al. [28] presented a lane-changing model to understand and predict the lane-changing behavior based on a game-theoretical approach that endogenously accounted for the flow of information in a connected vehicular environment. This could improve driving awareness of surrounding traffic conditions. Liu, et al. [29] modeled the vehicle interactions during merging process under an enhanced game-theoretic framework by adopting Nash equilibrium. And the testing results showed that this framework could achieve a relatively high accuracy of predicting vehicles' actions. In [30], the combination of interaction-aware intention estimation with maneuver-based motion prediction based on supervised learning was proposed and the motion intention was modeled by the game theoretic idea. However, uncertainties in games were seldom researched. And the payoff function in the game had not considered the prediction of traffic environments.

3 Methodologies

In this section, methodologies employed in this study will be presented, and the framework for maneuver prediction and planning will be introduced with details as shown in Fig. 3. By combining driving efficiency, preference as well as vehicle driving risk in the payoff function (outcomes of the actions or maneuvers), extensive form game theories are employed to model the interaction and gaming between multiple vehicles. The risk assessment in this study is on the basis of vehicle trajectory predicting under uncertainty which integrates dynamic and maneuver-based approaches. By solving the Nash equilibrium in mixed and behavioral strategies, the results of maneuver prediction of the surrounding vehicles and planning for the ego vehicle will be obtained.

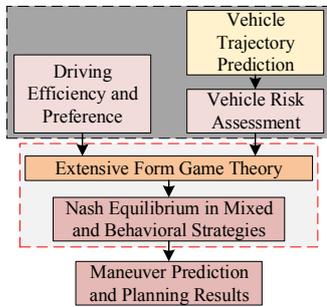


Fig. 3: The framework of maneuver prediction and planning

In this part, extensive form game theories including different Nash equilibriums will be introduced briefly. Then, a risk assessment model based on vehicle trajectory prediction under uncertainty is proposed and studied, which lays the foundation for the interaction and gaming between multiple vehicles. In the next subsection, the payoff function for interaction and gaming is designed by considering the collision risk, driving efficiency, and maneuver preference. Finally, the framework of maneuver prediction and planning will be proposed and presented based on the single-act and multi-act games in extensive forms.

3.1 Introduction of Extensive Form Game Theories

Game theories have been researched extensively and applied in different domains such as economics, politics, sociology, and military. In addition, game theories were employed in engineering fields including automated vehicles with the aim to understand and model

decision processes and SAs. In complex traffic environments, multiple agents (vehicles) make decisions in a situation where each other's payoff depends on the others' behaviors. Therefore, game theories could provide a promising framework for interaction and gaming awareness modeling [31]. According to the payoff function, the Nash equilibrium expresses the gaming results, which means that players could not do anything better with the consideration of the other players' actions or decisions.

To describe the interaction and gaming between multiple vehicles, non-cooperative game theories are used, in which each player makes decisions independently. The basic definition of the non-cooperative game theory is represented as follows:

- $P = \{P_1, P_2, \dots, P_i, \dots, P_N\}$ is the set of players, indexed by $i, i \in \{1, 2, \dots, N\}$;
- $A_i = \{A_i^1, A_i^2, \dots, A_i^k, \dots, A_i^M\}$ are the actions of the player P_i ;
- $U = \{U_1, U_2, \dots, U_i, \dots, U_N\}$ are the payoff functions for the players.

The problem of maneuver predicting and planning for a few steps in the near future is a multiple time-sequential problem, which could be described as dynamic games [32]. Dynamic games can be represented by a game tree, known as the extensive form games. Based on extensive form games, the interaction and gaming between multiple vehicles in the traffic could be simulated in a sequential manner.

The basic elements of the extensive form game can be expressed by a tuple as follows:

$$S = (T, P, A, U) \quad (1)$$

- T is a directed game tree with root and nodes;
- $P = \{P_1, P_2, \dots, P_i, \dots, P_N\}$ is the set of players, indexed by $i, i \in \{1, 2, \dots, N\}$;
- $A_i = \{A_i^1, A_i^2, \dots, A_i^k, \dots, A_i^M\}$ are the actions of the player P_i ;
- $A = \{A_1, A_2, \dots, A_k, \dots, A_N\}$ are the actions of the players;
- $U = \{U_1, U_2, \dots, U_i, \dots, U_N\}$ are the payoff functions for the players.

A game in an extensive form described by a tree includes nodes and edges. Nodes can be the player nodes or the end nodes. The end nodes are indicated by the outcomes of the sequential actions according to the payoff function. And each edge is an action of a player [33]. There are two kinds of extensive form game theories, namely perfect information, and imperfect information extensive games according to the agent's knowledge about all the prior choices including those of other agents. An imperfect-information game in extensive form is a game in which the players know about all the prior choices including those of other agents and each player's choice nodes are partitioned into information sets; definitely, if two choice nodes are in the same information set then the agent cannot distinguish between them [34]. In this study, the imperfect information extensive games are employed to model the interaction and gaming between multiple vehicles because some players could not know the actions of other players definitely at the same time.

There are all kinds of Nash equilibriums to describe gaming results in extensive forms. The basic equilibrium is called pure-strategy Nash equilibrium, which selects one of the available actions in each information set of that agent [32]. But in some cases, there are no pure strategies. As a result, Nash equilibrium in mixed and behavioral strategies is proposed to describe the gaming results via probability distributions. The mixed strategies for a player is a probability distribution on the set of all the player's pure strategies. On the other hand, behavioral strategies in the extensive form games are defined as a player denoting probability distributions only on the alternatives belonging to the information set.

Based on the extensive-form game theory, the maneuver prediction and planning in the traffic could be simulated. In the following subsections, the risk assessment is firstly introduced to evaluate collision risks of the traffic environment. Then, based on the risk

assessment, the pay-off function in the game theory is proposed, which considers the driving collision probability, driving efficiency, and driving preference. The final subsection is the introduction of maneuver prediction and planning with details.

3.2 Risk Assessment based on Vehicle Trajectory Prediction under Uncertainty

The common risk assessment models are based on defining risk functions according to dynamic features, such as time to collision (TTC), time to lane (TTL), and so on. These common risk assessment models are called dynamic feature based models. Dynamic feature based models project vehicle dynamic parameters such as the relative velocity, distance, and lateral acceleration for risk assessments. Always, machine learning methods such as support vector machine and decision trees were used extensively for projecting since they could deal with nonlinear problems with multiple features [18]. Moreover, Inverse reinforcement learning method was employed to learn the risk assessment model from driving demonstrations [35]. In addition, some complex mathematic models were proposed considering dynamic features such as the safety field model [36]. These models firstly define the relationship between features and calibrate the function parameters using realistic driving data [37]. However, these methods did not take future potential risks as well as uncertainties into consideration.

In this study, the risk assessment model proposed in this study is on the basis of vehicle trajectory prediction under uncertainty. As presented in Fig. 4, the vehicle trajectory prediction integrates the dynamic and maneuver-based approaches considering environmental uncertainties, which could predict accurately in the short term and look ahead to a high horizon [5]. Based on trajectory prediction, the collision at a specific time point could be estimated under uncertainty in the traffic environment [17]. Then, the risk assessment for a vehicle could be obtained by considering potential risks in the near future and combining lateral and longitudinal parameters between multiple vehicles [38].

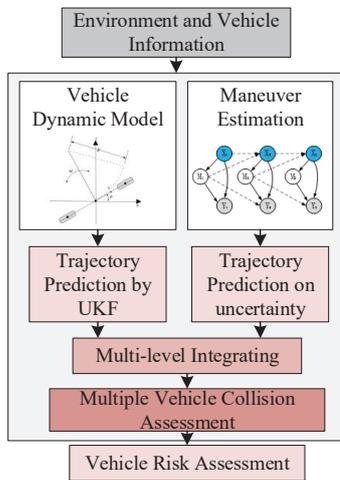


Fig. 4: Risk assessment model based on trajectory prediction under uncertainty

3.2.1 Collision Assessment at a Specific Time Point: Collision assessment based on trajectory prediction of two vehicles at a specific time point can be given as follows:

$$P(C_{v_i, v_j}(t)) = \iint C(x_{v_i}(t), x_{v_j}(t)) p(x_{v_i}(t), x_{v_j}(t)) dx_{v_i} dx_{v_j} t \in [t_0, t_0 + T_p] \quad (2)$$

where v_i presents the vehicle i , and t is the time. And $x_{v_i}(t)$ is the predicting position of v_i at time t . $p(x_{v_i}(t), x_{v_j}(t))$ is the position

probability of the vehicle i and j , which is introduced and researched in [5]. t_0 is the starting predicting point, and T_p is the predicting horizon time. $C(x_{v_i}(t), x_{v_j}(t))$ is the collision index as expressed as the following equation, which considers the shape of vehicles.

$$C(x_{v_i}(t), x_{v_j}(t)) = \begin{cases} 1, & V_i(x_{v_i}(t)) \cap V_j(x_{v_j}(t)) \neq \emptyset \\ 0, & \text{else} \end{cases} \quad (3)$$

where $V_i(x_{v_i}(t))$ means the space that the vehicle i covers.

In this study, the Monte Carlo (MC) method suggested in [27] has been used to calculate the collision assessment.

3.2.2 Risk Assessment: The risk assessment between two vehicles within the prediction horizon:

$$\text{Risk}(v_i(t_0 : t_0 + T_p), v_j(t_0 : t_0 + T_p)) \quad (4)$$

can be represented as the collision prediction distribution over the future time span.

$$\begin{aligned} \text{Risk}(v_i(t_0 : t_0 + T_p), v_j(t_0 : t_0 + T_p)) \\ = \int_{t_0}^{t_0 + T_{max}} P(C_{v_i, v_j}(t)) \frac{1}{t} dt \end{aligned} \quad (5)$$

where T_{max} safeties the following equation:

$$P(C_{v_i, v_j}(T_{max})) = \max_{t \in [t_0, t_0 + T_p]} P(C_{v_i, v_j}(t)) \quad (6)$$

In complex traffic scenarios such as the scenario shown in Fig. 2, the risk assessment should be considered in multiple vehicles. The risk assessment of the vehicle v_i in a scene s_i could be expressed as follows:

$$RA(v_i, s_i) = \max_j (\text{Risk}(v_i(t_0 : t_0 + T_p), v_j(t_0 : t_0 + T_p))) \quad (7)$$

where s_i presents the scene i , and v_i, v_j are the vehicles in the scene.

As shown above, the risk assessment model proposed in this study considers the uncertainty in the traffic environment and it is on the basis of traffic environment prediction, which considered future potential risks and could be applied in a variety of scenarios including intersection, lane changing, turning, and so on.

3.3 Payoff Function for Interaction and Gaming

The payoff function mentioned above is one of the core parts of the extensive form game theories to interpret the interaction and gaming between multiple vehicles. The payoff function designed in this study is able to assess the driving risk in the near future as well as driving efficiency. Also, in the payoff function, the driving preference is considered by adding the cost of lane switch and acceleration or deceleration, which aims to describe the preference of keeping in the original driving condition [40].

As a result, the payoff function is expressed as follows:

$$\begin{aligned} U_i = \lambda_1 RA(v_i, s_i) + \lambda_2 switch + \lambda_3 \frac{Acc}{\rho_v} \\ + \lambda_4 \int I(v_i, s_i, m_t^i) p_{pred}(\Delta v_{t_0 + T_p}, \Delta d_{t_0 + T_p}) dp \end{aligned} \quad (8)$$

where λ_i ($i = 1, 2, 3, 4$) present the coefficients, m_t^i means the maneuver that the vehicle takes, $switch$ is preference cost of changing lane, Acc is the absolute value of longitudinal acceleration. $\lambda_2 switch + \lambda_3 \frac{Acc}{\rho_v}$ represents the driving preference, which means the preference cost of changing lane and accelerating. ρ_v is the parameter for cost, $p_{pred}(v_{t_0 + T_p}, d_{t_0 + T_p})$ is the probabilistic distribution of the relative parameters at the end of prediction, and

$I(v_i, s_i, m_t^i)$ presents the driving efficiency for better traffic condition, which could assess the traffic beyond the prediction horizon and could be expressed as follows:

$$I(v_i, s_i, m_t^i) = \begin{cases} \frac{\Delta v_{t_0+T_p}}{\Delta d_{t_0+T_p}} & \text{if } \frac{\Delta v_{t_0+T_p}}{\Delta d_{t_0+T_p}} > 0; \\ 0 & \text{if } \frac{\Delta v_{t_0+T_p}}{\Delta d_{t_0+T_p}} \leq 0; \end{cases} \quad (9)$$

where $\Delta v_{t_0+T_p}$ is the relative speed between the ego vehicle and the vehicle ahead at time $t_0 + T_p$, and $\Delta d_{t_0+T_p}$ is the relative distance between the two vehicles at time $t_0 + T_p$. If there is no vehicle ahead, then $I(v_i, s_i, m_t^i) = 0$.

3.4 Maneuver Prediction and Planning Considering Interaction and Gaming

Based on the extensive form game theory with imperfect information, the maneuver prediction of other vehicles and the maneuver planning for the ego vehicle could be simulated and achieved.

In the extensive form game, define the vehicles in the traffic as players. And the ego vehicle is represented as P_0 , and other vehicles in the surrounding environment are defined as P_i ($i = 1, 2, \dots, N$). N is the number of the vehicles except P_0 .

Therefore, the players in the game can be represented as follows:

$$P = \{P_0, P_1, P_2, \dots, P_i, \dots, P_N\} \quad (10)$$

The actions $A_i = \{A_i^1, A_i^2, \dots, A_i^k, \dots, A_i^{M_i}\}$ for P_i in the game represent the maneuvers such as left lane-change, right lane-change, and lane keeping. M_i is the size of the actions for P_i .

In addition, $P_{-i} = [P_j]_{j \neq i}$ means vector of all players except P_i . In the same way, $a_{-i}^t(j) = [a_h^t(j)]_{h \neq i}, j \in M_i$ means vector of all action profiles for all players except P_i , where t means the time steps and $a_i^t(j) \in A_i$ means the j th maneuver at time t for player i . In other words, $a_0^1(j)$ is the planning maneuver for ego vehicle and $a_{-0}^1(j)$ is the predicted maneuvers of the other vehicles in the surrounding traffic at the first simulation time step.

Scenes of the environment are defined as $S = [S_1, S_2, \dots, S_i, \dots, S_m]$, where m is the size of the scenes. And $s_t \in S$ can be defined as follows:

$$s_t = [a_0^t(j), a_{-0}^t(j)] \quad (11)$$

where t is the simulation time step.

At the initial step $t = 0$, the maneuvers of surrounding vehicles are estimated based on the observable parameters using dynamic Bayesian networks studied in details in [46], in which the observable parameters are steering angle and lateral velocity using communication technologies. The probabilistic distribution of the maneuvers could be obtained and expressed as $p(a_i^0(j))$. Assume that the estimated maneuvers are independent as in [41]. Therefore, the probabilistic distribution of s_0 , $p(s_0)$ could be obtained as follows:

$$p(s_0) = \prod_{i=0}^n p(a_i^0(j)) \quad (12)$$

At step t , the probabilistic distribution of maneuvers for P_i could be expressed as $p(a_i^t(j))$, $j \in M_i$.

The uncertainty of the maneuver estimation is considered in the payoff function of the extensive form game, as shown in (2) and (8).

3.4.1 Single-step prediction and planning: The single-step prediction and planning are based on the single-act extensive game theory. In the single-act extensive game theory, players just game with each other based on a single step. In other words, each player

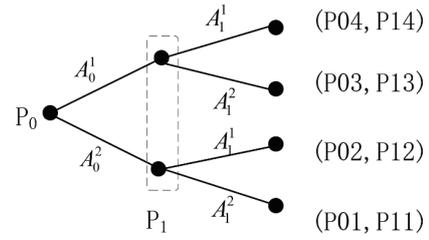


Fig. 5: The single-step prediction and planning based on an extensive game with two players. P_0, P_1 are the two players in this game. A_i^j is the action of P_i . P_{ij} is the payoff for Player i .

acts only once [42]. The example of a single-act extensive game with two players is shown in Fig. 5.

In the single-act extensive game theory, the payoff function is shown in (8), which is on the basis of the integrated trajectory prediction under uncertainty. As shown in Fig. 5, P_{ij} is the payoff for Player i . P_0, P_1 are the two players in this game. A_i^j is the action of P_i . In the same dotted area (known as the information set) shown in this figure, there are two possible nodes of P_1 , which means P_1 does not have access to the actions of P_0 even though P_1 acts firstly. In other words, this game is equivalent to the case when both players act simultaneously [42].

The maneuver prediction of the other vehicles and maneuver planning for the ego vehicle can be obtained by solving the Nash equilibrium in mixed and behavioral strategies. According to conclusions from [30], this imperfect games in extensive form could be translated into norm form games. And in the norm form game, mixed strategies could be solved as the results for the maneuver prediction and planning.

Let the notation Θ_i denote the mixed-strategy space for P_i . And $0 \leq \theta_i^j \leq 1$ represents the j th element of the space Θ_i , which means $\theta_i^j \in \Theta_i, j \in M_i$ and M_i means the size of the space Θ_i . Let $\{\theta_i^{j*} \in \Theta_i, i \in N, j \in M_i\}$ be a mixed-strategy noncooperative Nash equilibrium solution for this game if the following inequalities are satisfied for all $\theta_i^j \in \Theta_i, (j \in M_i, i \in N)$:

$$\begin{aligned} J_i^* &\triangleq \sum_{n_0=1}^{M_1} \dots \sum_{n_N=1}^{M_N} \theta_0^{n_0*} \theta_1^{n_1*} \theta_2^{n_2*} \dots \theta_i^{n_i*} \dots \theta_{N-1}^{n_{N-1}*} \theta_N^{n_N*} \\ &U_i(A_0^{n_0}, A_1^{n_1}, \dots, A_{N-1}^{n_{N-1}}, A_N^{n_N}) \\ &\leq \sum_{n_0=1}^{M_1} \dots \sum_{n_N=1}^{M_N} \theta_0^{n_0*} \theta_1^{n_1*} \theta_2^{n_2*} \dots \theta_i^{n_i} \dots \theta_{N-1}^{n_{N-1}*} \theta_N^{n_N*} \\ &U_i(A_0^{n_0}, A_1^{n_1}, \dots, A_{N-1}^{n_{N-1}}, A_N^{n_N}), \forall i \in N \end{aligned} \quad (13)$$

where $\{J_0^*, J_1^*, \dots, J_n^*\}$ is known as a noncooperative Nash equilibrium outcomes of the game in mixed strategies. These inequalities indicate that the vehicles in the surrounding traffic environment try to minimize their mixed payoff with the consideration of other vehicles' actions. Moreover, every game in the type of this study admits a noncooperative Nash equilibrium solution in mixed strategies [42].

Therefore, the maneuver planning of the ego vehicle P_0 for the single step could be obtained as follows:

$$A_0^* = A_0^{mx} \quad (14)$$

where mx satisfies $\theta_0^{mx*} = \max_{j \in M_0} \theta_0^{j*}$.

Moreover, the initial predicted maneuver distribution of P_i , $p(a_i^0(j))$ ($0 < j \leq M_i$) and the mixed-strategy noncooperative Nash equilibrium solution for interaction and gaming, $\{\theta_i^{1*}, \dots, \theta_i^{j*} \dots \theta_i^{M_i*}\}$ could be viewed as the index to evaluate

the predicting probabilities of maneuvers. As a result, the predicted maneuver distribution of P_i could be expressed as follows:

$$p(a_i^1(j)) = \frac{p(a_i^0(j)) + \theta_i^{j*}}{\sum_{h=1}^{M_i} (p(a_i^0(h)) + \theta_i^{h*})} \quad (15)$$

However, the single-step prediction and planning could only predict and plan for one time step, which means that it could not react earlier to potential risks or better traffic environment in the next few steps. In the next subsection, the multi-step prediction and planning would be introduced to solve the problem by employing multi-act extensive games.

3.4.2 Multi-step prediction and planning: Multi-step prediction and planning in this study are proposed to predict the environment and plan maneuvers for AVs by looking ahead in a few steps, which makes AVs react earlier to potential risks or better traffic environment in the future. Multi-step prediction and planning are based on multi-act games in extensive forms. In multi-act games, all players are allowed to act more than once in the game tree. As shown in Fig. 6, it is a 2-step prediction and planning tree, which has two players P_0 and P_1 . And each player acts twice in the game.

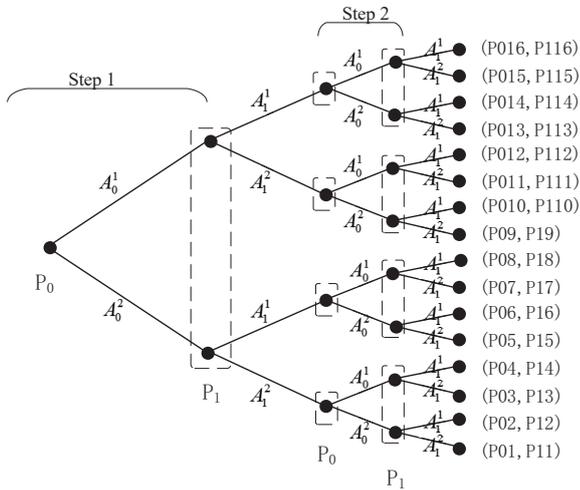


Fig. 6: Multi-step prediction and planning based on multi-act games in extensive forms with two players. P_0, P_1 are the two players in this game. A_i^j is the action of P_i . Pij is the payoff for Player i .

In the framework of multi-step prediction and planning, it is assumed that the players in the next step know exactly the actions in former steps. As shown in Fig. 6, the dashed curves of P_1 in Step-1 represents that P_1 in Step-1 do not know the action of P_0 in Step-1, which is same with the single-step prediction and planning tree. However, the players P_0, P_1 in Step-2 know exactly the actions in Step-1, which is shown by the information sets in Step-2.

The payoff costs of the multi-act games in extensive forms are defined by considering the discounted payoff of each step based on (8), which is expressed as the following equation:

$$MU_i = \sum_{k=1}^n \beta^{k-1} U_i^k, \quad 0 < \beta \leq 1 \quad (16)$$

where MU_i means the multi-step payoff of P_i . n is number of step, for example, in Fig. 6, the number of step $n = 2$. β is the discount of the payoff at every step. And U_i^k is the payoff from (8) at Step k . The multi-step payoff function indicates that the multi-act games in extensive forms include the influences from the next several steps.

The maneuver prediction of the other vehicles and maneuver planning for ego vehicle within several time steps can be obtained by

solving the Nash equilibrium in behavioral strategies. Let n denotes the number of levels of the play, then the behavioral strategies of P_i could be decomposed into n components $\hat{\gamma}_i^1, \hat{\gamma}_i^2, \dots, \hat{\gamma}_i^n$, where $\hat{\gamma}_i^k$ the corresponding behavioral strategy of P_i at its k th step of play. $\hat{\gamma}_i^k$ could be expressed as follows:

$$\hat{\gamma}_i^k = \{\theta_i^{k1}, \theta_i^{k2}, \dots, \theta_i^{kj}, \dots, \theta_i^{kM_i}\} \quad (17)$$

where k means the k th step of play, M_i is the size of the maneuver vector for Player i , and θ_i^{kj} means the probability of the maneuver j at the k th step of play for Player i .

Then the set of inequalities to determine the behavioral Nash equilibrium solution can be written. For $\forall 0 \leq i \leq N, 0 \leq k \leq n$, the inequalities (18) should be satisfied.

In this study, the Nash equilibrium in behavioral strategies of multi-act games with extensive forms can be obtained by solving a sequence of single-act games and by appropriate concatenation of the equilibrium strategies determined at each step of play [42]. Therefore, we could impose further restrictions that it also satisfies the inequalities (19) recursively at step k for all $\hat{\gamma}_i^k, \forall 0 \leq i \leq N, 0 \leq k \leq n$.

As a result, the maneuver planning for the ego vehicle could be achieved by finding the maximum probability behavioral strategy in the Nash equilibrium, expressed as follows:

$$A_0^{k*} = A_0^{kmx}, \quad 0 < k \leq n \quad (19)$$

where m_x satisfies $\theta_0^{kmx*} = \max_{j \in M_0} \theta_0^{kj*}, 0 < k \leq n$.

Moreover, the maneuver prediction for multiple steps could be obtained according to the Nash equilibrium in behavioral strategies and the former prediction results as shown in (20).

$$p(a_i^{k+1}(j)) = \frac{p(a_i^k(j)) + \theta_i^{kj*}}{\sum_{h=1}^{M_i} (p(a_i^k(h)) + \theta_i^{kh*})} \quad (20)$$

4 Application and Result Analysis

This section provides some application results in exact traffic scenarios and the results are analyzed to prove this framework using Matlab simulation, which can predict and plan for several time steps and drive cooperatively with rationality. In this part, the application scenarios will be described and the results based on the proposed framework are shown in this section.

4.1 Application Scenarios and Definition of Maneuvers

In this study, the prediction and planning framework has been applied and proved in different lane-change scenarios, where there are multiple vehicles in the surrounding environment in a three-lane road for one direction shown in Fig. 7.

As shown in the lane-change scenarios, maneuvers used in this application including lateral and longitudinal maneuvers. The lateral maneuvers A_{lat} are defined as follows:

$$A_{lat} = \{LLC, RLC, LK\} \quad (21)$$

where LLC means left lane-change, RLC means right lane-change, and LK represents lane keeping.

Meanwhile, the longitudinal maneuvers A_{long} could be defined as follows:

$$A_{long} = \{Dec, Const, Acc\} \quad (22)$$

where Dec means deceleration, $Const$ means keeping constant velocity, and Acc means acceleration.

As a result, the maneuvers of P_i in the surrounding traffic can be obtained:

$$A_i = A_{lat} \times A_{long} \quad (23)$$

The maneuver prediction and planning in different lane-change scenarios will be analyzed in the next subsection based on the basic maneuver definition in [43].

$$\begin{aligned}
J_i^* &\triangleq J_i \left(\hat{\gamma}_0^{1*}, \hat{\gamma}_0^{2*}, \dots, \hat{\gamma}_0^{k*}, \dots, \hat{\gamma}_0^{n*}; \dots; \hat{\gamma}_i^1, \hat{\gamma}_i^2, \dots, \hat{\gamma}_i^{k*}, \dots, \hat{\gamma}_i^{n*}; \dots; \hat{\gamma}_N^1, \hat{\gamma}_N^2, \dots, \hat{\gamma}_N^{k*}, \dots, \hat{\gamma}_N^{n*} \right) \\
&\leq J_i \left(\hat{\gamma}_0^1, \hat{\gamma}_0^2, \dots, \hat{\gamma}_0^k, \dots, \hat{\gamma}_0^n; \dots; \hat{\gamma}_i^1, \hat{\gamma}_i^2, \dots, \hat{\gamma}_i^k, \dots, \hat{\gamma}_i^n; \dots; \hat{\gamma}_N^1, \hat{\gamma}_N^2, \dots, \hat{\gamma}_N^k, \dots, \hat{\gamma}_N^n \right)
\end{aligned} \tag{18}$$

where J_i is the expected payoff based on the corresponding behavioral strategies. And $\{J_0^*, J_1^*, \dots, J_i^*, \dots, J_{N-1}^*, J_N^*\}$ is the noncooperative Nash equilibrium outcomes of the game in behavioral strategies.

$$\text{Step } k \left\{ \begin{aligned}
&J_0^* \triangleq J_0 \left(\hat{\gamma}_0^1, \hat{\gamma}_0^2, \dots, \hat{\gamma}_0^{k*}, \dots, \hat{\gamma}_0^{S*}; \dots; \hat{\gamma}_i^1, \hat{\gamma}_i^2, \dots, \hat{\gamma}_i^{k*}, \dots, \hat{\gamma}_i^{S*}; \dots; \hat{\gamma}_N^1, \hat{\gamma}_N^2, \dots, \hat{\gamma}_N^{k*}, \dots, \hat{\gamma}_N^{S*} \right) \\
&\leq J_i \left(\hat{\gamma}_0^1, \hat{\gamma}_0^2, \dots, \hat{\gamma}_0^k, \dots, \hat{\gamma}_0^{S*}; \dots; \hat{\gamma}_i^1, \hat{\gamma}_i^2, \dots, \hat{\gamma}_i^{k*}, \dots, \hat{\gamma}_i^{S*}; \dots; \hat{\gamma}_N^1, \hat{\gamma}_N^2, \dots, \hat{\gamma}_N^{k*}, \dots, \hat{\gamma}_N^{S*} \right) \\
&\dots \\
&J_i^* \triangleq J_i \left(\hat{\gamma}_0^1, \hat{\gamma}_0^2, \dots, \hat{\gamma}_0^{k*}, \dots, \hat{\gamma}_0^{S*}; \dots; \hat{\gamma}_i^1, \hat{\gamma}_i^2, \dots, \hat{\gamma}_i^{k*}, \dots, \hat{\gamma}_i^{S*}; \dots; \hat{\gamma}_N^1, \hat{\gamma}_N^2, \dots, \hat{\gamma}_N^{k*}, \dots, \hat{\gamma}_N^{S*} \right) \\
&\leq J_i \left(\hat{\gamma}_0^1, \hat{\gamma}_0^2, \dots, \hat{\gamma}_0^k, \dots, \hat{\gamma}_0^{S*}; \dots; \hat{\gamma}_i^1, \hat{\gamma}_i^2, \dots, \hat{\gamma}_i^{k*}, \dots, \hat{\gamma}_i^{S*}; \dots; \hat{\gamma}_N^1, \hat{\gamma}_N^2, \dots, \hat{\gamma}_N^{k*}, \dots, \hat{\gamma}_N^{S*} \right) \\
&\dots \\
&J_N^* \triangleq J_N \left(\hat{\gamma}_0^1, \hat{\gamma}_0^2, \dots, \hat{\gamma}_0^{k*}, \dots, \hat{\gamma}_0^{S*}; \dots; \hat{\gamma}_i^1, \hat{\gamma}_i^2, \dots, \hat{\gamma}_i^{k*}, \dots, \hat{\gamma}_i^{S*}; \dots; \hat{\gamma}_N^1, \hat{\gamma}_N^2, \dots, \hat{\gamma}_N^{k*}, \dots, \hat{\gamma}_N^{S*} \right) \\
&\leq J_i \left(\hat{\gamma}_0^1, \hat{\gamma}_0^2, \dots, \hat{\gamma}_0^k, \dots, \hat{\gamma}_0^{S*}; \dots; \hat{\gamma}_i^1, \hat{\gamma}_i^2, \dots, \hat{\gamma}_i^{k*}, \dots, \hat{\gamma}_i^{S*}; \dots; \hat{\gamma}_N^1, \hat{\gamma}_N^2, \dots, \hat{\gamma}_N^k, \dots, \hat{\gamma}_N^{S*} \right)
\end{aligned} \right. \tag{19}$$

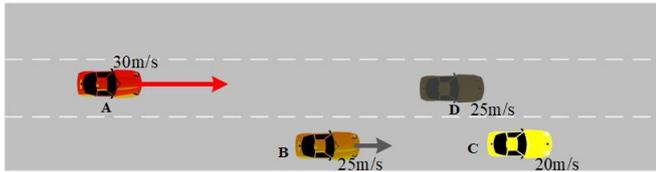


Fig. 7: Multiple vehicles in a three-lane road for one direction

4.2 Result Analysis

The results of the maneuver prediction and planning framework proposed in this study will be analyzed via three different lane-change scenarios. In each one, the specific lane-change scenario will be introduced first and then the results will be presented and analyzed. In each scenario, it is assumed that the ego vehicle interacts and games with the vehicle just near ahead in the three lanes [28]. And $\lambda_i = 1$ ($i = 1, 2, 3, 4$), $\beta = 0.8$, which means that the payoff elements have the same weight and the discount of the payoff with the time step is 0.8.

4.2.1 First lane-change scenario: The first lane-change scenario is shown in Fig. 8. In this traffic scenario, there are three vehicles, namely Vehicle A, Vehicle B, and Vehicle C. And Vehicle A is the ego vehicle, which means that the maneuver planning results of Vehicle A and the maneuver prediction of Vehicle B would be obtained in this application considering interaction and gaming. Other vehicles are assumed to keep their maneuvers. For example, Vehicle C keeps its maneuver (LK) based on the observe-based dynamic Bayesian network model introduced with details in [46], which could use the detecting parameters such as the lateral velocity only to predict other vehicles' maneuvers.

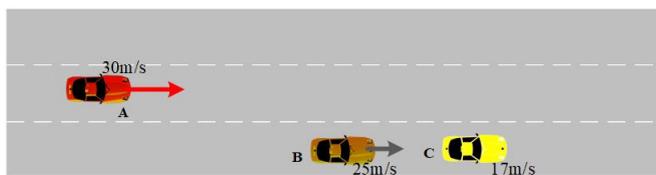


Fig. 8: The first lane-change scenario for application.

In this scenario, Vehicle B with an initial speed of 25 m/s and C with the speed of 17 m/s are in the first lane. And Vehicle B

is approaching Vehicle C with higher velocity. Vehicle A with the initial speed of 30 m/s is in the middle lane. The relative longitudinal distance between Vehicle A and Vehicle B is 30 m and that between Vehicle B and Vehicle C is 20 m. It is expected that Vehicle B may have the high intention or probability to make the left lane-change for better traffic, even with low probability of the left lane-change according to the observe-based model. And by considering this, Vehicle A should make the left lane-change in case that Vehicle B would merge in.

The results are shown in Table 1. As shown in the table, the maneuver planning for Vehicle A is $\{LLC, Const\}$ according to the single-step prediction and planning method, which means Vehicle A should make the left lane-change with the constant velocity. And Vehicle B has high probability of making the left lane-change (with the probability of 0.53) by considering the multiple vehicles' interaction and gaming. In the aspect of maneuver prediction, the probability of LK for Vehicle B is 0.95 and that of LLC is 0.05 according to the observe-based model with only the dynamic parameters. However, the probability of LLC for Vehicle B is much higher if considering the interaction and gaming between multiple vehicles.

If the relative longitudinal distance between Vehicle A and Vehicle B becomes longer. For example, the relative longitudinal distance between Vehicle A and Vehicle B is 35 m and that between Vehicle B and Vehicle C is 20 m. Then the maneuver planning for Vehicle A according to single-step prediction and planning is $A_0^1 = LK, Dec$, but the maneuver planning for Vehicle A is $A_0^1 = \{LLC, Const\}$ and $A_0^2 = \{LLC, Const\}$ according to the multiple-step prediction and planning method. This indicates that the multiple-step prediction and planning could know the potential risks and make the cooperative decision ahead.

Table 1: Maneuver prediction and planning results for the first scenario

	Vehicle A (Maneuver Planning)	Vehicle B (Lateral Maneuver Prediction)
Observe-based Model	No Planning Results	$p(A_1^0 = LLC) = 0.05$ $p(A_1^0 = RLC) = 0$ $p(A_1^0 = LK) = 0.95$
Single-step prediction and Planning	$A_0^1 = \{LLC, Const\}$	$p(A_1^1 = LLC) = 0.53$ $p(A_1^1 = RLC) = 0$ $p(A_1^1 = LK) = 0.47$
2-step Prediction and Planning	$A_0^1 = \{LLC, Const\}$ $A_0^2 = \{LLC, Const\}$	$p(A_1^2 = LLC) = 0.77$ $p(A_1^2 = RLC) = 0$ $p(A_1^2 = LK) = 0.23$

The results indicate that the proposed framework could predict the maneuvers of surrounding vehicles considering the interaction and gaming. Also, the maneuver planning results for the ego vehicle show that this framework could make the decisions with cooperation by considering other vehicles' strategies. Moreover, results indicate that multiple step prediction and planning framework could make the ego vehicle be aware of the potential risk and make the cooperative decisions earlier.

4.2.2 Second lane-change scenario: The second lane-change scenario is shown in Fig. 9. In this scenario, there are four vehicles and it is assumed that Vehicle A is the ego vehicle. In this case, there is another more vehicle (Vehicle D) also in the middle lane with slow velocity (18 m/s) compared with the first scenario. The relative longitudinal distance between Vehicle A and Vehicle B is 30 m and that between Vehicle B and Vehicle C is 20 m. And the relative longitudinal distance between Vehicle B and Vehicle D is 10 m. It is expected that Vehicle B may have the high intention or probability to make lane keeping because there is a slow vehicle in the middle lane. And Vehicle A should make the left lane-change for better traffic environment when approaching the slow vehicle D.

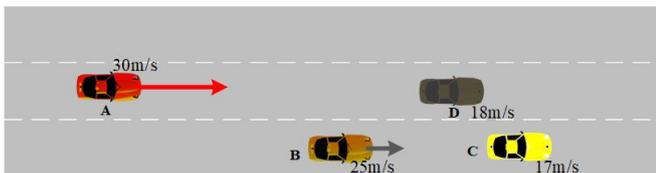


Fig. 9: The second lane-change scenario for application.

In this scenario, the maneuver prediction and planning results are shown in Table 2.

Table 2 Maneuver prediction and planning results for the second scenario

	Vehicle A (Maneuver Planning)	Vehicle B (Lateral Maneuver Prediction)
Observe-based Model	No Planning Results	$p(A_1^0 = LLC) = 0.05$ $p(A_1^0 = RLC) = 0$ $p(A_1^0 = LK) = 0.95$
Single-step prediction and Planning	$A_0^1 = \{LLC, Const\}$	$p(A_1^1 = LLC) = 0.02$ $p(A_1^1 = RLC) = 0$ $p(A_1^1 = LK) = 0.98$
2-step Prediction and Planning	$A_0^1 = \{LLC, Const\}$ $A_0^2 = \{LLC, Const\}$	$p(A_1^2 = LLC) = 0.01$ $p(A_1^2 = RLC) = 0$ $p(A_1^2 = LK) = 0.99$

As shown in Table 2, the maneuver planning for Vehicle A is $\{LLC, Const\}$ in both the single-step and multiple-step prediction and planning methods, which could obtain better traffic environment in the third lane because of the slow vehicle ahead in the middle lane. In this case, for the reason that there is a slow vehicle ahead in the middle lane, Vehicle B is predicted with higher probability of lane keeping than that of the observe-based model by considering the interaction and gaming aspect.

4.2.3 Third lane-change scenario: The third lane-change scenario is shown in Fig. 10. In this scenario, there are four vehicles and it is assumed that Vehicle A is the ego vehicle. In this case, there is another vehicle (Vehicle D) in the third lane compared with the first scenario. The velocity of Vehicle D is 33 m/s. The relative longitudinal distance between Vehicle A and Vehicle B is 30 m and that between Vehicle B and Vehicle C is 20 m. And the relative longitudinal distance between Vehicle A and Vehicle D is 5 m. It is expected that Vehicle B may have the high intention or probability to make lane changing for better traffic environment. Even being awareness of this, Vehicle A should keep its lane and decelerate because of the

approaching of vehicle D as well as in the case of the left lane-change for Vehicle B.

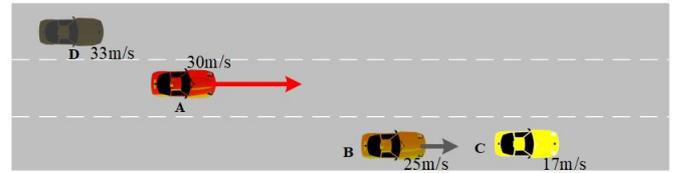


Fig. 10: The third lane-change scenario for application.

In this scenario, as shown in Table 3, the maneuver prediction of Vehicle B is similar to that in the first scenario by considering the interaction and gaming with Vehicle A. However, the maneuver planning of Vehicle A is $\{LK, Dec\}$, which means Vehicle A should stay in the middle lane because Vehicle D is approaching Vehicle A with higher velocity. Also, since there is a high probability that Vehicle B would merge in, Vehicle A has to decelerate at the same time.

Table 3 Maneuver prediction and planning results for the third scenario

	Vehicle A (Maneuver Planning)	Vehicle B (Lateral Maneuver Prediction)
Observe-based Model	No Planning Results	$p(A_1^0 = LLC) = 0.05$ $p(A_1^0 = RLC) = 0$ $p(A_1^0 = LK) = 0.95$
Single-step prediction and Planning	$A_0^1 = \{LK, Dec\}$	$p(A_1^1 = LLC) = 0.53$ $p(A_1^1 = RLC) = 0$ $p(A_1^1 = LK) = 0.47$
2-step Prediction and Planning	$A_0^1 = \{LK, Dec\}$ $A_0^2 = \{LK, Dec\}$	$p(A_1^2 = LLC) = 0.77$ $p(A_1^2 = RLC) = 0$ $p(A_1^2 = LK) = 0.23$

In this study, the alpha-beta pruning is employed to eliminate the possible maneuvers in the extensive game tree [43]. The average computing time of the framework running on an Intel Core i7-800GB at 2.2GHz is 85 milliseconds, which proves an online maneuver prediction and planning. The parallel computing or distributed algorithm could be used to decrease the computing time in the future work.

5 Conclusions and Contributions

In this study, a framework of maneuver prediction and planning is proposed on the basis of interaction and gaming between multiple vehicles. The payoff costs of multiple vehicles include risk assessments as well as the driving efficiency and preference. The risk assessment model proposed in this study is on the basis of trajectory prediction under uncertainty which integrates the dynamic and maneuver-based models for more accurate prediction in the long term. Based on the extensive-form game theory, the single-step and multi-step maneuver prediction and planning frameworks are proposed. And the maneuver predicting results combine observe-based model as well as the Nash equilibrium in mixed and behavioral strategies. In addition, this framework is proved and applied in different lane-change scenarios. The results indicate that the maneuver prediction of vehicles considers the interaction and gaming between multiple vehicles. Moreover, the planning results for the ego vehicle show the cooperative decision-making strategies, which consider other vehicles strategies at the same time. Also, the multi-step prediction and planning makes the decision strategies being aware of the potential risks and respond earlier.

In this study, the maneuver prediction and planning results are based on the extensive-form game theories. In the future work, multiple agent reinforcement learning methods could be employed to

learn and optimize the decision making by considering other agents' strategies and driver characteristics, which could learn and optimize the system itself [45][47]. Also, the parallel computing and distributed algorithms will be used to improve the computing time when more vehicles are considered and more steps are needed.

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