B&B Algorithm-Based Green Light Optimal Speed Advisory Applied to Contiguous Intersections

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Abstract

Signalized intersections play important roles in transportation efficiency and vehicle fuel consumption. The upcoming signal information obtained by DSRC communication can be used for green light optimal speed advisory (GLOSA) to design the optimal velocity profile of approaching vehicles, which benefits fuel economy and traffic efficiency. In this paper, the branch and bound (B&B) algorithm based GLOSA applying to contiguous intersections is developed, aiming to improve the fuel economy. Using a communication platform, field tests were conducted to verify the GLOSA algorithm. The tests results indicate that, in free-flow conditions, the GLOSA algorithm can decrease stop times and fuel consumption without sacrificing trip efficiency.

Keywords: GLOSA; B&B algorithm; Contiguous intersections; Fuel consumption.

INTRODUCTION

With the improvement of the economy, there were more than1 billion vehicles on the road all over the world up to the end of 2010(Sousanis, 2011). Along with a rapidly increasing number of global vehicles, the energy consumed by vehicles is mushrooming, and the traffic situations in urban areas become more aggravated.

The increasing number of vehicles exceeds the capacity of urban roads, which is the main cause of traffic congestion. Traffic jams, however, cannot be solved by simply increasing the number of road and widening roads because of high cost and long construction periods. As an important and effective solution, road efficiency should be improved (Wen, 2008). The vital factor of urban road efficiency is signalized intersections. A vehicle will consume fuel in great quantities when idling at signalized intersections. According to the *Urban Mobility Report 2010*, idling vehicles consume more than2.8 billion gallons of gasoline(Lomax et al, 2010).Based on the above, it is obvious that signalized intersections play important roles in transportation efficiency and vehicle fuel consumption.

To improve traffic efficiency at signalized intersections, much research has been carried out on the optimization of traffic signal phasing and timing (Wolshon et al., 1999; Han, 1996; Hunt et al., 1982). According to the research, traffic signals are adjusted according to real traffic situations. The methods above work well on improvement of traffic efficiency. As another important part of road traffic, vehicles can also adapt their motions to traffic signals. V2X communication provides an opportunity to obtain richer information, which contributes to more efficient and safer traffic (Amanna, 2009; Zheng et al., 2014 & 2015; Stubing et al., 2010; Lu et al., 2010). Upcoming traffic signal information obtained by V2X communication can be used for green light optimal speed advisory (GLOSA) to design the optimal velocity profile of approaching vehicles, which benefits fuel economy and traffic efficiency.

The main contributions of this paper are as follows:

- Developing a GLOSA algorithm to apply to contiguous intersections, which optimizes the velocity profile of the approaching vehicle, aimed at fuel economy and trip efficiency.
- Setting up a test bed and validating the effectiveness of the GLOSA algorithm through field tests.

The rest of this paper organized as follows: The following section discuss related work. Methodology explains the optimization model of GLOSA and presents the branch and bound (B&B) algorithm to solve the optimal problem. The field test section introduces the field tests, followed by results and discussion. The Conclusion section concludes this paper.

RELATED WORK

Researchers have made great efforts to reduce vehicles fuel consumption at intersections or improve traffic efficiency with V2X communication or ad vehicular ad hoc networks (VANET) technology.

Alsabaan et al. (2013) developed a comprehensive optimization model that involved V2V and V2I communication to minimize fuel consumption and emissions. The authors proposed a geocast protocol that formulated how traffic light signals transmit the signal phase and time (SPaT) information to vehicles and how the vehicles exchanged the SPaT information. A heuristic expression was proposed to compute the optimum or near-optimum velocity.

Asadi and Vahidi (2011) presented an optimization-based control algorithm that used upcoming traffic signal information within the vehicle's adaptive cruise control system to reduce idle time and fuel consumption at stop lines.

Li et al. (2014) considered fuel consumption and developed a GLOSA algorithm. When a vehicle approaches the intersection, the deceleration, deceleration time, acceleration, and acceleration time were optimized using a Lagrangian Genetic Algorithm. Seredynski et al. (2013) introduced a multi-segment GLOSA according to which several lights in sequence on a vehicle's route are taken into account. The optimization speed was computed using a genetic algorithm. After simulation, it was found that, in free-flow conditions, the multi-segment GLOSA provides much better results when compared with a single-segment approach.

Some research has studied the performance of GLOSA. Katsaros et al. (2011) monitored the impacts of GLOSA on fuel and traffic efficiency by introducing metrics for average fuel consumption and average stop time behind a traffic light using a simulation method. The simulation result proved that GLOSA systems could improve fuel consumption and reduce traffic congestion at junctions.

The aforementioned studies mainly differ from this paper in the optimization model and the method of GLOSA; they do not prove the effectiveness of the GLOSA by field tests. First, we built a kinematics model for vehicles passing contiguous intersections, and then a mathematic optimization problem was built to minimize total fuel consumption. Also, a test bed was developed and the field tests were conducted to validate the effectiveness of the GLOSA algorithm proposed.

METHODOLOGY

Kinematic model

When approaching an intersection, a vehicle will adjust its velocity and then pass the intersection with constant velocity. It is assumed that in the adjustment process the vehicle is running with constant acceleration or deceleration. As soon as the vehicle passes the intersection, it will start to adjust the velocity again. In Figure 1, as an example, when approaching contiguous intersections, the vehicle adapts its velocity to target velocity v_i and keeps the pace until passing each intersection.

According to traffic laws, the vehicle must pass the intersection during green light duration. Here, it is supposed that the vehicle is not allowed pass the stop line of the intersection during the red phase and the yellow phase. In Figure 2, the red and yellow phases are combined, as shown with red lines; the green phase is shown with green lines. The variables, g_{ij} and r_{ij} shown in Figure 2, determine the signal phases of the contiguous intersections. The variables, d_i , shown in Figure 2, determine the road geometry.

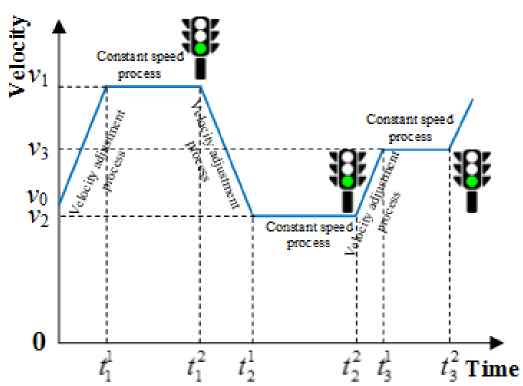

Figure 1. Velocity profile

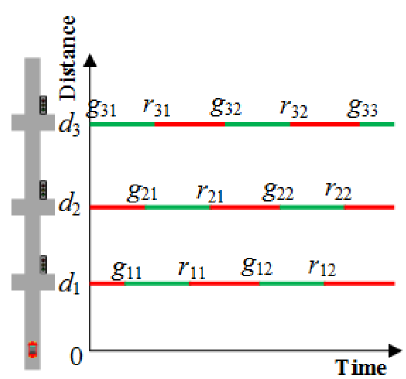

Figure 2. Signal phases

Based on the assumptions above, some constraints must be satisfied, including kinematic constraints, green light constraints, velocity limit constraints, and acceleration constraints. Notations used here are defined in Table 1.

Table 1. Definition of Notations

v_0	Initial velocity
v_i	Target velocity in the *i*-th segment
t_i^1	Moment when vehicles start to run with constant speed in the *i*-th segment
t_i^2	Moment when the vehicle is passing the stop line in the *i*-th segment
d_i	Distance between the start point and the *i*-th intersection
a_i	Acceleration during the velocity adjustment process in the *i*-th segment
N	Number of intersections
g_{ij}	Moment of the *j*-th green phase starting at the *i*-th intersection
r_{ij}	Moment of the *j*-th green phase ending at the *i*-th intersection
$v_{\min}$	Minimum velocity of the vehicle
$v_{\max}$	Maximum velocity allowed by traffic laws or the traffic condition
a_{acc}	Constant acceleration when the vehicle accelerates
a_{dec}	Constant acceleration when the vehicle decelerates

(1) Kinematic constraints

$$d_1 = v_0 t_1^1 + 0.5 a_1 \left(t_1^1\right)^2 + v_1 \left(t_1^2 - t_1^1\right) \tag{1}$$

$$d_{i+1} - d_i = v_i \left(t_{i+1}^1 - t_i^2\right) + 0.5 a_{i+1} \left(t_{i+1}^1 - t_i^2\right)^2 + v_{i+1} \left(t_{i+1}^2 - t_{i+1}^1\right) \quad i = 1,2,\cdots,N \tag{2}$$

The kinematic constraints specify the mathematical relationships among the velocity, trip time, and distance of the vehicle.

(2) Green light constraints

$$t_i^2 \in \bigcup_{j \in N^*} \left[g_{ij}, r_{ij}\right] \tag{3}$$

The green light constraints guarantee that the vehicle obeys the traffic signals.

(3) Velocity limit constraints

$$v_i \in \left[v_{\min}, v_{\max}\right] \tag{5}$$

(4) Acceleration constraints

$$a_i \in \left\{a_{\text{acc}}, a_{\text{dec}}\right\} \tag{6}$$

Here, it is assumed that the vehicle accelerates/decelerates with the same and certain acceleration/deceleration during each velocity adjustment process.

Fuel consumption model

Much research has studied the fuel consumption model. The power-based fuel consumption model was built by Post as early as 1984 (Post et al., 1984). Akcelik (1989), An et al. (1993) and Wong (2011) improved the power-based fuel consumption. Rakha et al. (2011) proposed the power-based and torque-based polynomial fuel consumption model and the evaluation criteria. In this paper, the 4-parameter polynomial fuel consumption model was chosen, which is shown in equation (7). Notations used here are defined in Table 2.

$$F_C = \alpha_1 + \alpha_2\omega_e + \alpha_3\omega_e T_e + \alpha_4\omega_e^2 T_e \quad T_e > 0 \tag{7}$$

The engine torque T_e can be calculated as follows without considering the rotational inertia of the tires and flywheel.

$$T_e = \frac{r_w}{i_T\eta}\left(\frac{1}{2}C_D A\rho_a v^2 + Gf\cos\alpha + G\sin\alpha + \frac{G}{g}\frac{dv}{dt}\right) \tag{8}$$

The vehicle velocity can be calculated using rotation speed of the engine.

$$v = \frac{r_w}{i_T}\omega_e \tag{9}$$

Based on equation (7~9), the mathematic relationship among the fuel consumption, velocity, and acceleration is built as follows according to equation (10).

$$F_c = k_0 + \left(k_1 + k_2\frac{dv}{dt}\right)v + \left(k_3 + k_4\frac{dv}{dt}\right)v^2 + k_5 v^3 + k_6 v^4 \ , T_e > 0 \tag{10}$$

Here, the coefficients, $k_0 \sim k_6$, need to be fitted based on the fuel consumption data.

Table 2. Definition of Notations

F_C	Fuel consumption rate
$\alpha_1 \sim \alpha_4$	Coefficients of the polynomial fuel consumption model
ω_e	Rotation speed of the engine
T_e	Brake torque of the engine
r_w	Radius of the tire
i_T	Transmission ratio from the engine to the tire
C_D	Drag coefficient
A	Windward area
ρ_a	Air density
v	Vehicle velocity
m	Vehicle mass
α	Ramp angle
$k_0 \sim k_6$	Coefficients of the fuel consumption model

Optimization Model

In this paper, as the objective function, the total fuel consumption to pass N intersections is minimized. Under the constraints above, the objective function is to be optimized to find the optimal velocity to pass each intersection. This optimization model is written as expression (11).

Here, $F(v_i)$ refers to the total fuel consumption indicated as equation (12).

$$
\begin{aligned}
&\min_{v_i} F\left(v_i\right) \quad i=1,2,\cdots,N \\
&s.t. \\
&\begin{cases}
d_1 = v_0 t_1^1 + 0.5 a_1 \left(t_1^1\right)^2 + v_1\left(t_1^2 - t_1^1\right) \\
d_{i+1} - d_i = v_i\left(t_{i+1}^1 - t_i^2\right) + 0.5 a_{i+1}\left(t_{i+1}^1 - t_i^2\right)^2 + v_{i+1}\left(t_{i+1}^2 - t_{i+1}^1\right) \\
t_i^2 \in \bigcup_{j\in N^*}\left[g_{ij}, r_{ij}\right] \\
v_i \in \left[v_{\min}, v_{\max}\right] \\
a_i \in \left\{a_{\text{acc}}, a_{\text{dec}}\right\}
\end{cases}
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
F\left(v_i\right) = &\sum_{i=1}^{N}\int_{t_{i-1}^2}^{t_i^1}\left(k_0 + \left(k_1 + k_2 a_i\right)v + \left(k_3 + k_4 a_i\right)v^2 + k_5 v^3 + k_6 v^4\right)dt \\
&+\sum_{i=1}^{N}\int_{t_i^1}^{t_i^2}\left(k_0 + k_1 v_i + k_3 v_i^2 + k_5 v_i^3 + k_6 v_i^4\right)dt \quad \left(t_0^2 = 0\right)
\end{aligned}
\tag{12}
$$

B&B algorithm

The optimization problem above is a non-convex and nonlinear problem with the non-convex and nonlinear objective function and the non-convex and nonlinear feasible region. It is difficult to find an analytical solution. In this paper, the B&B algorithm is used to obtain the approximate optimal solution.

The first step of the B&B algorithm is to discretize the target velocity in each segment according to the velocity limit constraints. The discretization step should be selected with the tradeoff between accuracy and instantaneity. The discretization values of the target velocity in each segment named nodes constitute a solution space tree with plenty of branches, as Figure 3 shows.

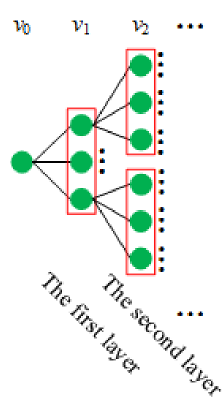

Figure 3. Solution space tree

Then, the corresponding trip time of each node in the first layer is calculated using the kinematic constraints. When the trip time fails to reach the green light constraint at the first intersection, the corresponding branch should be cut. Otherwise, go to the next step. Next, the corresponding trip time of each node in the i-th layer is calculated using the kinematic constraints. When the trip time fails to reach the green light constraint at the *i*-th intersection, the corresponding branch should be cut. Otherwise, reserve the branch. After the feasible solution space is found, the value of the objective function is computed and the optimal solution is sought out.

A block diagram and a schematic diagram are shown in Figure 4 and Figure 5, respectively.

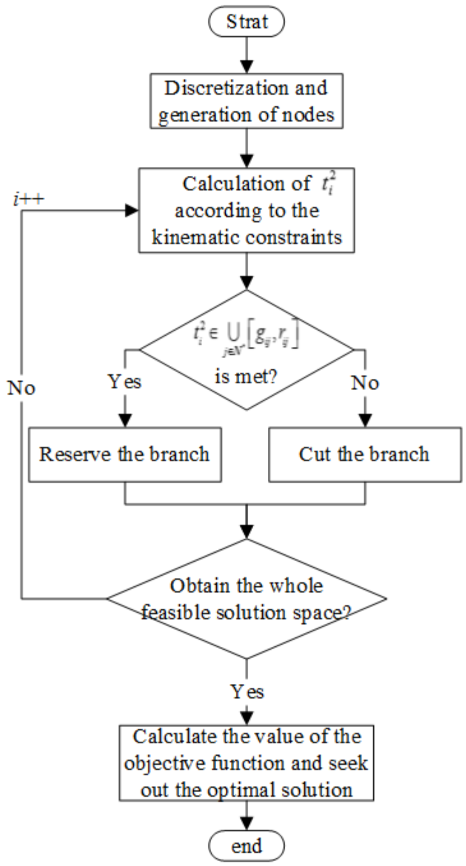

Figure 4. Block diagram

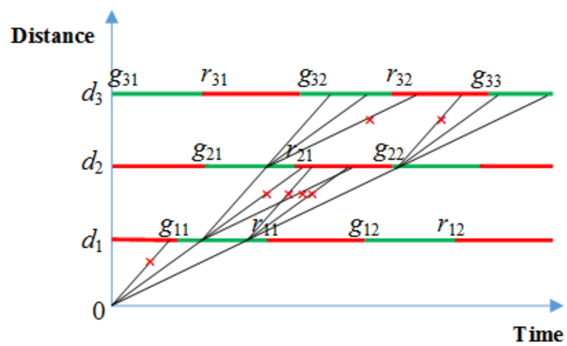

Figure 5. Schematic diagram

FIELD TEST

Test bed

The test bed, on basis of three contiguous intersections, was built in Beijing. The test bed on the roadside collected SPaT information and sent it to vehicles by DSRC. The equipment at every intersection was connected by an Ethernet network, and the SPaT information was shared so that the vehicle could receive the SPaT information of three intersections in the whole area. The distance between the first intersection and the second intersection was 300m and between the second intersection and the third intersection was 200m.

The geometry and schematic diagram of the test bed are shown in Figure 6.

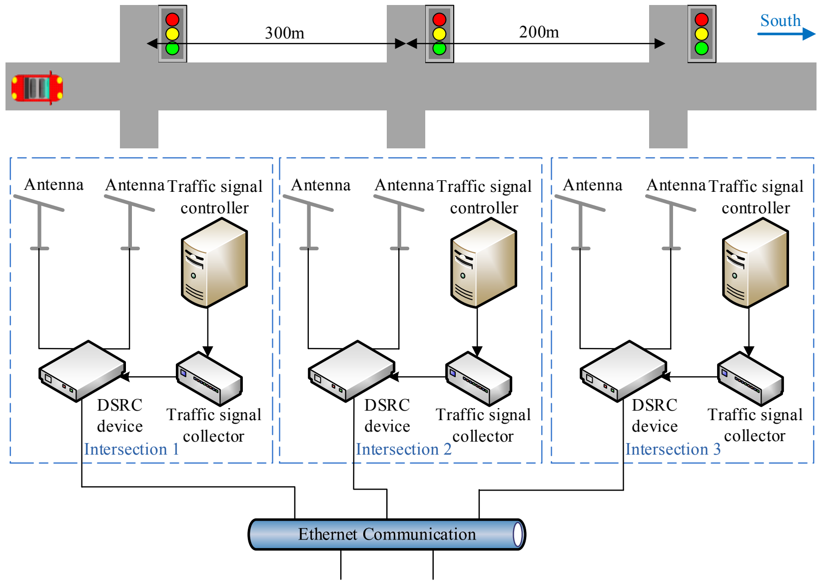

Figure 6. Schematic diagram of test bed

Experiment vehicle platform

The experiment vehicle platform could receive the information from roadside units by DSRC, collect the vehicle status information, locate the vehicle position, calculate the target velocity, and give the driver a prompt by HMI. Figure 7 shows the physical map of the experiment vehicle platform.

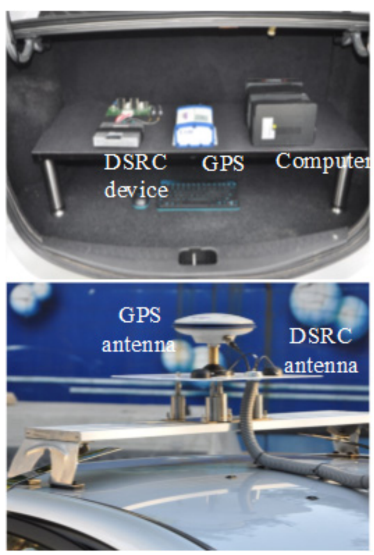

Figure 7. Physical map of platform

Test method

Five drivers were invited to drive the vehicle to pass the three contiguous intersections. In one scenario, the driver drove the vehicle freely. In the other scenario, the GLOSA calculated the target velocity and gave it to the driver by audible prompts. To avoid the interference of other vehicles, the field tests were conducted in late night, which means that the algorithm was applied in free-flow conditions.

The velocity profiles, position profiles, and fuel consumption were collected. It is worth noting that the regions of interest included the areas from a point 100m before the first intersection to a point 50m after the last intersection. The fuel consumption mentioned below refers to the total fuel consumption in the regions of interest. The regions of interest are shown in Figure 8.

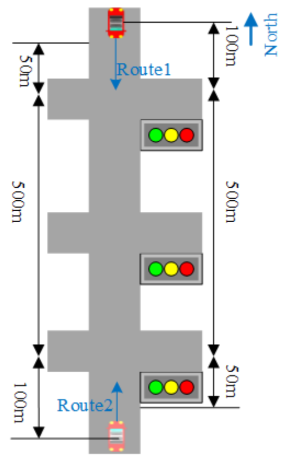

Figure 8. Regions of interest

RESULTS AND DISCUSSION

Results

The average fuel consumption, total stop times, and average trip time of different drivers in different scenarios were statistically analyzed. Table 3 and Table 4 show the statistical results.

Table 3. Average Fuel Consumption (mm^3)

Driver ID	Freely driving	With prompt	Improvement
1	73280	60920	16.9%
2	72820	58336	19.9%
3	69320	65655	5.3%
4	80392	64672	19.6%
5	72920	63368	13.1%
Average	73746	62590	15.1%

Table 4. Average Trip Time and Total Stop Times

Driver ID	Average trip time(s)		Total stop times	
	Freely driving	With prompt	Freely driving	With prompt
1	125	104	18	11
2	119	113	21	16
3	113	131	20	16
4	130	127	24	16
5	119	121	22	16
Average	121	119	21	15

The data show that the GLOSA improved fuel economy by an average of 15.1%, and the improvement reached approximate 20% for drivers 2 and 4. The GLOSA increased trip time by an average of 2 s, which is not statistically significant. With the GLOSA, total stop times decreased greatly. In conclusion, it is clear that the GLOSA developed in this paper can improve fuel economy and decrease stop times without affecting trip efficiency.

Discussion

The reasons for the improvement of fuel economy and the decrease of stop times is that with the goal of fuel economy, the GLOSA using the SPaT information provides drivers with the eco-velocity to avoid unnecessary stop and go and frequent acceleration and deceleration. Three typical scenarios were selected to make clear the reasons.

In the first scenario, shown in Figure 9, the vehicle is driving at an excessive velocity without the GLOSA, which leads to the stop at the end of the red phase. With the prompt of the GLOSA, a driver will slow down and pass the intersection without stopping at the stop line.

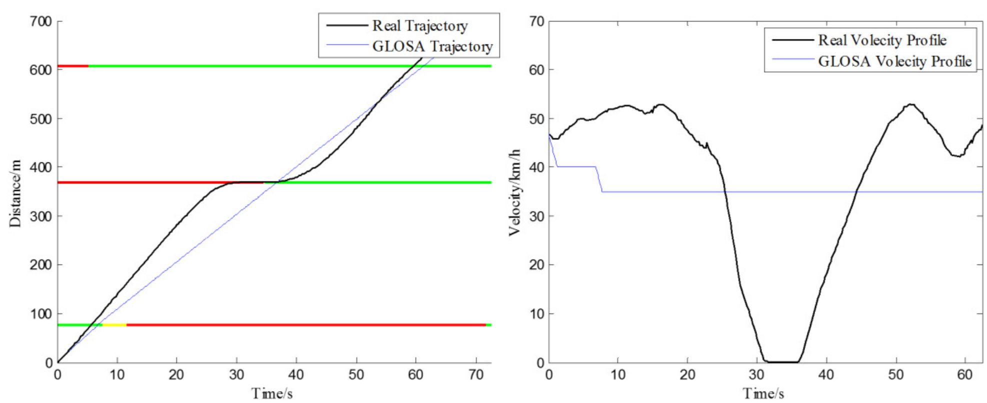

Figure 9. Distance trajectory and velocity profile in first scenario

In the second scenario, shown in Figure 10, the vehicle drives at a lower velocity without GLOSA, which results in losing the opportunity to pass the intersection at the end of the green phase. With the GLOSA, a driver will speed up and avoid stopping at the beginning of the red phase.

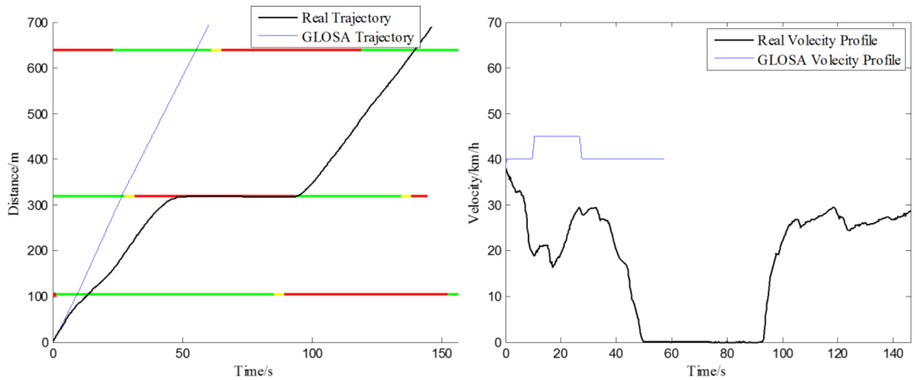

Figure 10. Distance trajectory and velocity profile in the second scenario

In the third scenario, shown in Figure 11, the driver speeds up when the signal phase remains green and then slows down when the signal phase switches to red, which causes fluctuating velocity and damages fuel economy. With the GLOSA, however, the driver will keep a relatively steady velocity.

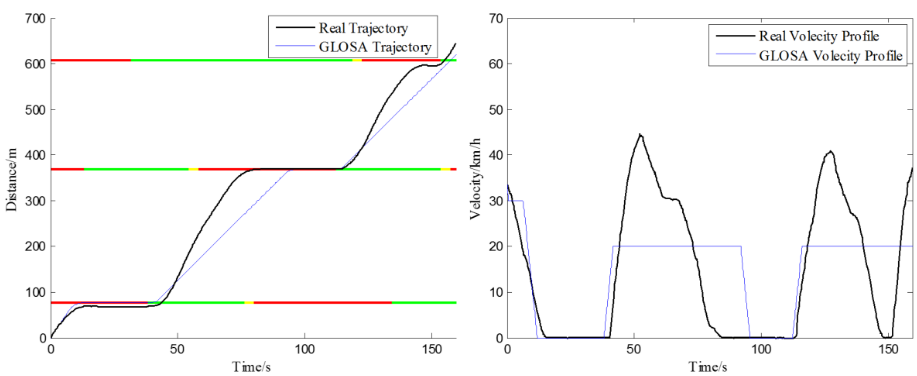

Figure 11. Distance trajectory and velocity profile in the third scenario

CONCLUSION

A B&B algorithm-based GLOSA applied to contiguous intersections has been developed in this paper. First, a kinematic model that describes the kinematic relationship when a vehicle passes contiguous intersections was built. Then, the optimization model with the objective function of fuel consumption was proposed, and the B&B algorithm was developed to solve the optimization problem. Field tests were conducted to validate the GLOSA algorithm. Test results show that in free-flow conditions, the GLOSA algorithm can decrease stop times and fuel consumption without affecting trip efficiency.

ACKNOWLEDGEMENTS

This research is funded by the Chinese National Program for High Technology Research and Development (No. 2014AA110302), which is also supported by the joint research project of Tsinghua University and NISSAN Motor, Co., Ltd..

REFERENCES

Akcelik, R. (1989). "Efficiency and drag in the power-based model of fuel consumption." *Transportation Research Part B*, 23(5), 376-385.

Alsabaan, M., Naik, K., and Khalifa, T. (2013). "Optimization of fuel cost and emissions using V2V communications." *IEEE Transactions on ITS*, 14(3), 1449-1461.

Amanna, A. (2009). "Overview of IntelliDrive/Vehicle Infrastructure Integration (VII)." Virginia Tech Transportation Institute.

An, F., and Ross, M. (1993). "A model of fuel economy and driving patterns." SAE Technical Paper.

Asadi, B., and Vahidi, A. (2011). "Predictive cruise control: Utilizing upcoming traffic signal information for improving fuel economy and reducing trip time." *IEEE Transactions on Control Systems Technology*, 19(3), 707-714.

Han, B. (1996). "Optimising traffic signal settings for periods of time-varying demand." *Transportation Research Part A*, 30(3), 207-230.

Hunt, P. B., and Robertson, D. I., et al. (1982). "The SCOOT on-line traffic signal optimisation technique." *Traffic Engineering & Control*, 23(4).

Katsaros, K., Kernchen, R., et al. (2011, July). "Performance study of a Green Light Optimized Speed Advisory (GLOSA) application using an integrated cooperative ITS simulation platform." *Proceedings of 7th IEEE* International *Wireless Communications and Mobile Computing Conference*, 918-923.

Li, J., Dridi, M., and El-Moudni, A. (2014). "Multi-vehicles green light optimal speed advisory based on the augmented lagrangian genetic algorithm." *Proceedings of 17th IEEE Intelligent Transportation Systems Conference*, 2434-2439.

Lomax, T., Schrank, D., and Turner, S. (2010). "Urban Mobility Report 2010." *Texas Transportation Institute. http://mobility.tamu.edu/ums*.

Lu, X. Y., Wang, J., Li, S. E., and Zheng, Y. (2014). "Multiple-vehicle longitudinal collision mitigation by coordinated brake control." *Mathematical Problems in Engineering*, *2014*.

Post, K., Kent, J. H., Tomlin, J., and Carruthers, N. (1984). "Fuel consumption and emission modelling by power demand and a comparison with other models." *Transportation Research Part A*, 18(3), 191-213.

Rakha, H. A., Ahn, K., Moran, K., Saerens, B., and Bulck, E. V. D. (2011). "Virginia Tech comprehensive power-based fuel consumption model: Model development and testing." *Transportation Research Part D*, 16(7), 492-503.

Seredynski, M., Mazurczyk, W., et al. (2013). "Multi-segment green light optimal speed advisory." 27th IEEE International Parallel and Distributed Processing Symposium Workshops & PhD Forum (IPDPSW), 459-465.

Sousanis, J. (2011). "World vehicle population tops 1 billion units." WardsAuto.com, 15 Aug. 2009; http://wardsauto.com/ar/world_vehicle_population_110815.

Stubing, H., Bechler, M., et al. (2010). "Sim TD: A car-to-X system architecture for field operational tests [Topics in Automotive Networking]." *Communications Magazine, IEEE*, 48(5), 148-154.

Wen, W. (2008). "A dynamic and automatic traffic light control expert system for solving the road congestion problem." *Expert Systems with Applications*, 34(4), 2370-2381.

Wolshon, B., and Taylor, W. C. (1999). "Analysis of intersection delay under real-time adaptive signal control." *Transportation Research Part C*, 7(1), 53-72.

Wong, J. Y. (2001). *Theory of Ground Vehicles*. John Wiley and Sons.

Zheng, Y., Li, S. E., et al. (2014). "Influence of information flow topology on closed-loop stability of vehicle platoon with rigid formation." *Proceedings of the 17th IEEE Intelligent Transportation Systems Conference*, 2094-2100.

Zheng Y., Li S., Wang J., Cao D., and Li K. (2015) "Stability and scalability of homogeneous vehicular platoon: Study on influence of information flow topologies," *Intelligent Transportation Systems, IEEE Transactions on*.